

Analyse of the Linear Double Discrete Systems

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Abstract: The aim of this investigation is incorporate the approach in the design of complex discrete system, It also aims at presenting the some definitions and examples. In this study, a brief overview of the notion of discrete models is given.

Key words: Discrete systems

INTRODUCTION

Although their behaviour is containly ultimately continuous, such systems are most of the time operating in a discrete fashion. This means that their behaviour can be faithfully abstracted by a succession of steady states intermixed with jumps which make their state suddenly changing to others. Of course, the number of such possible changes is enormous and they are occuring in a cocurrent fashion at an unthinkable frequency, but this number and this high frequency do not change the very nature of the problem such systems are intrinsically discrete. They fall under the genetic name of transition systems. Having sard this does not make us moving very much towards a methodology, but it gives us at a common point of departure. Some of the examples envisaged above are pure programs, what ever the agent that executes them. The electronic circuit and sarting program clearly fall into this category: the to unsitions are essentially concentrated in one medium; the silicium or the computer. Most of the other examples however are far more complex than just pure programs because they involve many different executing agents and also a heavy interactions with their environment. But again, this does not change the very discrete system nature of the problem, it only complicates matters.

The purpose of this study is to approach in the design of complex discrete systems.

Discrete Transfer Solution: The input and output behaviour of discrete systems were invastigated by using by using the Z-transform is similar to the role of a place in analysis. There fore, it provides a convenuent way to establish the input and output of discrete time systems.

The Z-Transform: Let $\eta(k)$ be a time signal of the discrete system. The Z-transform of $\eta(k)$, denoted by

$X(Z)$, is defined to be:

$$Z \{ \eta(k) \} := X(Z) = \sum_{k=-\infty}^{\infty} \eta(k) Z^{-k}, \quad r_0 < |Z| < R_0 \quad (1)$$

Where $r_0 < |Z| < R_0$ is the region of convergence for the series in (1)

Example: Let $\eta(k) = b^k l(k)$, where $l(k)$ is the discrete time step function:

$$l(k) = \begin{cases} 1 & ; k \geq 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

Find $X(Z)$ the Z-transform of $\eta(k)$.

Solution: $X(Z) = z / z-b, \quad |Z| > |b|$

Properties of Z-transform: Let f_k and g_k be a discrete-time signals, Z-transforms $F(z)$ and $G(z)$, respectively

- Linearity: for real numbers a and b ,

$$Z \{ a f_k + b g_k \} = a Z \{ f_k \} + b Z \{ g_k \}$$

- Convolution of time sequences. Define the convolution of f_k and g_k to be:

$$(f * g)_k = \sum_{l=-\infty}^{\infty} f_l g_{k-l}$$

in this situation,

$$Z \{ (f * g)_k \} = Z \{ f_k \} Z \{ g_k \}$$

- Time shift. For every integer n ,

$$Z \{ f_{k+n} \} = z^n Z \{ f_k \}$$

- Scaling in the Z-plane. For any nonzero real number r ,

$$Z\{r^{-k} f_k\} = F(z)$$

- Final value theorem. When the limits exist

$$\lim_{k \rightarrow \infty} f_k = \lim_{x \rightarrow 1} (Z-1) F(Z)$$

- Inversion. We say f_k is one-sided when $f_k=0$ for $k<0$. When f_k is one-sided, we have:

$$F(Z) = \sum_{k=0}^{\infty} f_k Z^{-k}$$

Observe that when is the Z-transform of a one-sided discrete-time signal, its region of convergence has to be the form $r_0 < |Z|$

For one sided transfer functions it may be possible to unite $F(Z)$ as a rational function in Z^{-1} and expand it into partial fractions. The inverse Z-transform of $F(Z)$ would then be the sum of inverses of all partial fractions in the expansion of $F(Z)$, which could be easily looked up from a table of elementary Z-transforms.

Example : find u_k , where:

$$U(z) = 1 + z^{-1} / (1 - Z^{-1})(1 - 0.5Z^{-1})$$

$$\text{Solution: } u_k = (k - 3 / 2^k) \ell(k)$$

Discrete system: The foundation of these systems happen the algebra functions. The variations characterized of these systems are three items.

- Input variables: These are happened by the mean system and are effect the examined system.
- Output variables: These are characterized the properties of the examined system.
- Interval variables: These contain another variables which have been different than the input and the output variables.

System: Figure 1.1. the shape of the input and the output variables.

In this system, there are the input symbols in U numbers and output symbols in W number. Asll of them are interested to the period. That is $x(i)(t)$, $i=1, 2, 3, \dots, u$ is shown the input symbols and $Z(i)(t)$, $j=1, 2, 3, \dots, w$ is shown the output symbols.

Definition: It have been defined as the discrete system $X = \{\xi_1, \xi_2, \dots, \xi_p\}$ input alphabe and $Z = \{\eta_1, \eta_2, \dots, \eta_q\}$ output alphabe and below f_z and f_s charecteristic function system.

There, $S = \{\sigma_1, \sigma_2, \dots, \sigma_n\}$ formations are together the set and $Z_v = f_z(Z_v, S_v)$, $S_{v+1} = f_s(x_v, S_v)$

There, it have been known the in put and the output symbols and the system formation, in period. In this

situation, these charecteristic functions are defined for every (x_v, S_v) ,

If this equal is $f_z(x_v, S_v) \neq f_z(x_v)$, these systems are defined as trivial system. In contrast to this, when it is $f_z(x_v, S_v) \neq f(x_v)$, this system is defined as untrivial system.

Definition: It is defined as the input series to the input symbol series $\xi_{i2}, \dots, \xi_{iL}$ come immediately after ξ_{i1} input symbol and it is shown as $\xi_{i1}, \xi_{i2}, \dots, \xi_{iL}$ similarly the output serials are defined, also. In series, serial logitude converius to the contained. The effect and the esistanze are certain move in the disarete system and these are in the shape input and output series. If the longitude of the input series is L, the longitude of the output series is L, also. The situation in the begining of each system is important in the investigation of each system and this is defined as the begining property.

Theorem: We assurue that, the characteristic functions have been given as fz and fs and untrivial M system. That time, M system in them begining of σ_{i0} , the effect to the $\xi_{j1}, \xi_{j2}, \dots, \xi_{jL}$ input series:

- It is not possible before to determine , if only fz and fs given.
- It is fz, fs and σ_{i0} known, it is not possible to write.

Proof:

- M system is not triual system, according to the establish, the most less so σ_u and σ_w situations and ξ_h symbol are finding that

$$f_z(\xi_h, \sigma_u) \neq (\xi_h, \sigma_v)$$

In this time, the resistances according to the $\xi_h, \xi_{j2}, \xi_{jL}$ input series of M system $\sigma_{i0} = \sigma_u$ and if $\sigma_{i0} = \sigma_v$ is, that is different.

So, we can say before the resistance for a in put series, if it is not only in which the charecteristic functions of only fz and fs are certain.

- If fs and σ_{i0} are given in this time, ξ_{jk} situation of M system and the effect of $\xi_{j1}, \xi_{j2}, \dots, \xi_{jk}$ input series are happen the certain. If it is $K=0$, it is true because of σ_{i0} is certain if we will be accept it is true for K, in this case the true of that is find and when fs is given, σ_{ik} is certain. So σ_{ik+1} is obtain from $\sigma_{ik+1} = fs(\xi_{jk}, \sigma_{ik})$. In addiotion to this if fz function has been given, this time. η_{hk} input series is find by from $\eta_{hk} = fz(\xi_{jk}, \sigma_{ik})$. We can know before the $\xi_{h1}, \xi_{h2}, \xi_{hL}$ output series for $\xi_{j1}, \xi_{j2}, \xi_{jL}$ output series when fz and fs and σ_0 are given.

Passing Table (Transition Table): The equations of discrete system above and fz and fs functions can give with the passing table. In this table, the values according all possibility chanes of the characteristic functions, that is, all (X_v, S_v) values are written. There, X_v is component

Table 1: Passing table

S _v	X _v	Z _v				S _{v+1}			
		ξ ₁	ξ ₂	...	ξ ₃	ξ ₁	ξ ₂	...	ξ ₃
σ ₁		{η ₁ , η ₂ , ..., η _q } in squares and the values in it's the heap are written				{σ ₁ , σ ₂ , ..., σ _q } in squares and the values in it's the heap are written.			
σ ₂		{η ₁ , η ₂ , ..., η _q }				{σ ₁ , σ ₂ , ..., σ _q }			

of the in put alphabe and S_v is component of the situation alphabet

The example of the passing table is given for the finute system happen the situation heap {σ₁, σ₂, ..., σ_n}, the input alphabet {ξ₁, ξ₂, ..., ξ_p} and the output alphabet {η₁, η₂, ..., η_p}. In this table, there are Zu subtable and S_{v+1} subtable suitable for fz and fs charecteristic functions, intervals. The heads in these two subdepartments are same. So that, at that moment there are the suitable departments for each X_v symbols in departments. Thus all vertical departments are shown {ξ₁, ξ₂, ..., ξ_p} and all horizontal lines are shown {σ₁, σ₂, ..., σ_n}. Fz (ξ_j, σ_i) value (we will establish as Z_v this value) in Z subtable of ξ_j vertical line with σ_i line and fs (ξ_j, σ_i) value in S_{v+1} subtable (We will establish as S_{v+1} this value) are written. These are compoments which these symbols wrote in Z_v and S_{v+1} subtables are componens took from Z alphabet and S situations heap.

In general the multiparameters discrete systems are given with below equations:

$$\xi_v(S(c) = F_v(c, S(c), x(c)) \quad v=1, \dots, k, [GF(2)]^m \quad (1.1)$$

$$S(c^0) = S^0$$

Linear Double Discrete Systems: In general, below functions are given in Linear double discrete system:

$$Z_v = f(X_{v-1}, X_{v-2}, X_{v-i_u}, Z_{v-1}, Z_{v-2}, Z_{v-j_v})$$

$$X_{v-1} + X_{v-2} + \dots + X_{v-i_u} + Z_{v-1} + Z_{v-2} + \dots + Z_{v-j_v}$$

Where that, 0 ≤ i₁ < i₂ < i_n and 1 ≤ j₁ < j₂ < j_v
The essential case for the linear double automat

$$X_{v-1} = X_{v-2} = \dots = X_{v-i_u} = Z_{v-1} = Z_{v-2} = \dots = Z_{v-j_v} = 0$$

Now let's give V late and V operatör:

$$\Delta^r_y = y\gamma - r, \quad r=0, 1, 2, \dots$$

Where that, it can be both Y and Z. We will write Y instead of Δ⁰. (1,1) is Linear automat can write as below by the latring operatör.

$$IZ = \Delta^{i1}X + \Delta^{i2}X + \dots + \Delta^{iv}X + \Delta^{i1}Z + \Delta^{i2}Z + \dots + \Delta^{iv}Z \quad (1.2)$$

If we will add Δⁱ¹Z + Δⁱ²Z + ... + Δ^{iv}Z (according to the modul Z) to each one of (1,2) equation we obtain below:

$$\hat{I}Z + \Delta^{i1}Z + \Delta^{i2}Z + \dots + \Delta^{iv}Z = \Delta^{i1}X + \Delta^{i2}X + \dots + \Delta^{iv}X$$

Or,

$$(\Delta^{iv} + \dots + \Delta^{i2} + \Delta^{i1} + \hat{I})Z = (\Delta^{iv} + \dots + \Delta^{i2} + \Delta^{i1})X$$

The determination of Linear double discrete system can establish with T giving behaviour:

$$\frac{\Delta^{iv} + \dots}{\Delta^{iv} + \dots}$$

As shown that, if the giving behaviour the concerning with Linear discrete systems is known clearly, in this situation every time numbered diversity above equations:

$$\Delta^r Z = \Delta^r (\Delta^{i1}X + \Delta^{i2}X + \dots + \Delta^{iv}X + \Delta^{i1}Z + \Delta^{i2}Z + \dots + \Delta^{iv}Z)$$

$$(\Delta K + \Delta K^{-1} + \dots + \hat{I})Z = (\Delta K + \Delta K^{-1} + \dots + \hat{I})$$

$$(\Delta^{i1}X + \Delta^{i2}X + \dots + \Delta^{iv}X + \Delta^{i1}Z + \Delta^{i2}Z + \dots + \Delta^{iv}Z)$$

If linear double discrete systems are essential, in these case, the linear double automat is named as the subnet case.

As saw above equations, if Linear double automat is subnet in the begining, that time, the apportoinments and the denominator of the giving manner may pulse with the random polinom.

For example, Let's care for below Linear double discrete systems:

$$Z_v = X_{v-1} + X_{v-3} + X_{v-5} + X_{v-6} + X_{v-7} + X_{v-8} + Z_{v-1} + Z_{v-6} + Z_{v-7}$$

Or

$$Z_{v-7} + Z_{v-6} + Z_{v-1} + Z_v = X_{v-8} + X_{v-7} + X_{v-6} + X_{v-5} + X_{v-3} + X_{v-1}$$

At the same time, if Linear discrete double system are sunnet at the begining fo i and the accomplice of the giving manner of the automant, in this situation, this part ner pulsing may hung on. Become shrink yo related the accomplice pulsing is very clear which this reduce the degrees of the numeration and the denominator in giving attitude and thus Linear discrete systems simplity the equation. The partner pulsing can find by benetiting from Eudiol algritma.

Let's carry out the accomplice pulsing of the numeration and denominator in the giving attitude or Euclid algorithm for find the most accomplice dividing.

Therefore, the automat in the beginning time and the subnet situation:

$$\begin{aligned} Z_v &= \dots(1,4) \text{ or} \\ X_{(v-4)} &+ \dots (1,5) \\ Z_v &= X_{(v-4)} + \dots \\ Z_v &= X_{(v-8)} + \dots + Z_{v-1} \end{aligned}$$

These equations can be writing for example, if the automat take charge of is subnet, then it is 011011010001 the output suitable to 100111001010 or in input line according to (6) equal.

If we calculate the output being equivalent to 100111001010 level of (1) automat, the result of this calculating is 011011010001.

$$\begin{aligned} &100111001010 \\ X_0 &= 1, Z_0 = 0 \\ X_1 &= 0, Z_1 = X_0 = 1 \\ X_2 &= 0, Z_2 = X_0 + X_1 = 1 \end{aligned}$$

Truly, it is approving for simplifying which the numeration and the denominator are divided by accomplice number, also. Thus Linear double automat simplified contain the heap been all possible situations in the beginning of it.

In practice, a lot of Linear double automat met is beginning situation and therefore so simplifies are true and the results wishing.

REFERENCES

1. Kucera, V., 1995. "Analysis and Design of Discrete Linear Control System", Prentice Hall, Englewood Cliffs.

2. Gill, A., 1969. "Linear Modular Systems" In: L.A. Zadeh, E. Polak, "System Theory", McGraw-Hill, New York.

3. Wolovich, W.A., 1974. "Linear Multivariable Systems", Springer, New York.

4. Kaczorek T. 1993. Linear Control Systems.-New York: Wiley

5. Klamka, J., 1988a. M-Dimensional nonstationary linear discrete system in Banach spaces. -Proc. 12-th IMACS World Congress, Paris, Vol 4, pp.31-33.

6. Klamka, J., 1995. Constrained controllability of nonlinear systems.-IMAJ. Math. Contr. Inf., Vol. 12, No.2, pp. 245-252.

7. Klamka, J., 2002. Controllability of Nonlinear Discrete Systems. Int. J. Appl. Math. Comput. Sci., Vol. 12, No. 2, 173-180.

8. Amir Beck and Marc Teboulle, 2000. Global Optimality Condition For Quadratic Optimization Problem with Binary Constraints, SIAM Journal optimization, vol. N 1, pp: 179-188.

9. Peng, Y., X. Gungming and W. Long, "controllability of linear discrete-time systems with time-delay in state", submitted to ACC03.

10. Kaileth, T., 1980. "Linear systems", Englewood Cliffs, N.J., prentice-hall.

11. Phat, V.N., 2001. "Euclidean space controllability of singularly perturbed linear systems with state delay", System and Control Letters, v 43, n 3, 181-191.

12. Lind, P.G., J. Corte-Real and J.A.C. Gallas., 2003. geophysical Research Abstracts, vol.5, 11115,

13. Abrial., J.R., 2003. Discrete System Models., April.